Quiz 5 practice – Continuous probability distributions

1. Given a random variable having the normal distribution with µ = 16.2 and σ2 = 1.3225, find the probabilities that it will take on a value
   1. greater than 16.8;
   2. less than 14.9
   3. between 13.6 and 18.8
   4. between 16.5 and 16.7
2. Human body temperatures are normally distributed with a mean of 98.20oF and a standard deviation of 0.62oF. A temperature greater than 100.6oF is considered to be a fever. What proportion of people have fevers?
3. Men’s heights are normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches. Women’s heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. The Boeing 757 airplane’s doors are 72 inches from top to bottom.
   1. What proportion of men can fit through the door without bending?
   2. What proportion of women can fit through the door without bending?
4. Birth weights in Norway are normally distributed with a mean of 3570g and a standard deviation of 500g.
   1. If a hospital requires special treatment for babies that weigh less than 2700g, what percentage of newborn babies require special treatment?
   2. If the hospital requires special treatment for the smallest 3% of babies, what birth weight separates babies that require special treatment from those that don’t?
5. In a certain antipsychotic medication (tablet form) the mass of the active ingredientis normally distributed with mean 51 mg and standard deviation 2.5 mg.
   1. If the rated content of active ingredient in the tablets is 50 mg then what percentage of these tablets will have less than the rated amount of active ingredient?
   2. Suppose a patient receives 10 such tablets. What is the probability that at least one of the tablets will have less than the rated amount of active ingredient? (Hint: does the binomial distribution apply here?)
   3. If the so-called acceptable limits of active ingredient are 47 mg to 55 mg then what percentage of tablets will lie outside the acceptable limits?
   4. It turns out that a simple adjustment on the machine used to manufacture the tablets allows one to change the mean content without changing the standard deviation. To what level (in mg) should the mean be raised in order that only 1% of the tablets will lie below the (lower) acceptable limit? At this setting for the mean, what percentage of tablets will lie above the (upper) acceptable limit?
   5. With some effort it is possible to reduce the standard deviation. With the mean set at 51 mg, to what value must the standard deviation be reduced in order that only 5% of all tablets will have a mass of active ingredient which is outside the acceptable limits quoted above?

1. Specifications for a certain job call for washers with an inside diameter of 0.300 ± 0.002”. If the inside diameters of the washers supplied by a given manufacture may be looked upon as a random variable having the normal distribution with µ = 0.301” and σ = 0.001”, what percentage of these washers will meet specifications?
2. The length of a structural component of a device is an approximately normally distributed random variable with a standard deviation of 0.90 mm. The fabricating machine can be adjusted to achieve any desired mean value. What must the mean value be so that 90% of the components have a length of 12.10 mm or greater?
3. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with mean 50 days. Find the probabilities that such a camera will
   1. have to be reset in less than 25 days
   2. not have to be reset in at least 65 days
   3. last between 60 and 80 days
4. The time between arrivals at a passport office is modelled by an exponential random variable with a rate of 0.2 per minute. Find the probabilities that the waiting time between customers will be:
   1. Less than 5 minutes
   2. Between 7 and 9 minutes
   3. More than 10 minutes
5. An air actuated electric switch has an exponential life distribution with mean of 1000 hours.
   1. What proportion of the devices last at least 1150 hours?
   2. Find the median lifetime of the devices. That is – find the lifetime such that 50% of devices last less than that amount of time.
6. An integrated circuit chip has an exponential failure rate of 0.048 per thousand hours. What is the probability that it will operate satisfactorily for at least 15000 hours?

**Answers:**

1. a) 0.301 b) 0.129 c) 0.976 d) 0.0652
2. 0.005%
3. a) 85.77% b) 99.96%
4. a) 4.09% b) 2630g
5. a) 0.3446 b) 0.9854 c) 10.96% d) 52.82; 19.12% e) 2.041 mg
6. 0.8400
7. 13.25mm
8. a) 0.3935 b) 0.2725 c) 0.09930
9. a) 0.6321 b) 0.08130 c) 0.1353
10. a) 0.3166 b) 693.1h
11. 0.4868